

## Determination of Earth Station Antenna G/T Using the Sun or the Moon as an RF Source

For Small Aperture ( $\leq 5.0\text{m}$ ) Parabolic Earth Station Antenna Systems Operating from L to Ka-Band

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### ABSTRACT

The need for reliable calibrated sources of RF emissions in determining the gain-to-noise temperature ratios (G/T) of Earth Station Antenna Systems is ever present in today's world of rapidly expanding satellite telecommunications systems. Historically the Sun has been the primary source of RF emissions for these measurements as its location is easily calculated, solar flux density data is measured daily in multiple locations, and it is a very strong source of RF emissions meaning smaller systems (5.0 m and below) are able to utilize this source. There are currently numerous algorithms that allow the use of the Sun as an RF source for measuring G/T on all types of antenna systems. The Sun however is not in all cases the most reliable or convenient source of RF emissions for these measurements. There exist cases in which other RF sources should be considered, such as extreme latitude locations on the Earth where the sun may not be visible for several weeks. In these situations, an alternative source of RF emissions must be used. Radio stars such as Cassiopeia or Taurus are commonly used for large systems, however for smaller systems, the  $y$  factor will not be high enough for a reliable calculation. This leaves the Moon as the primary celestial source of RF emissions for smaller antenna systems. This report discusses an algorithm that will allow smaller antenna systems to use the Moon as a source of RF emissions. The resulting G/T measurement is potentially more accurate than the typical Sun based G/T measurement due to the increased stability and predictability of Lunar Flux Density versus Solar Flux Density.

### INTRODUCTION

Antenna gain-to-noise-temperature (G/T) is a figure of merit with regards to the characterization of an antenna's performance, where  $G$  is the antenna gain in decibels at the receive frequency, and  $T$  is the equivalent noise temperature of the receiving system in Kelvin. There are two primary methods for obtaining an antenna's G/T parameter, a direct measurement or calculating a mathematical approximation. This paper will focus on the former.

To obtain an accurate measurement of antenna G/T a source of RF emissions is needed. Typical RF sources are either another antenna with known parameters or a celestial body such as the sun or a radio star. The use of celestial bodies, primarily the sun, is most common since these bodies are visible for extended periods of time at almost any location on Earth. A problem arises however in geographical locations where the sun may not be visible for several weeks (i.e. extreme latitude locations). These locations are ideal for LEO tracking. This is not a problem for larger systems ( $> 10\text{m}$ ) as there are several radio stars visible in each hemisphere (i.e. Cassiopeia A, Taurus A, Orion, Virgo, Omega) that have well defined flux density equations that can be used for an accurate

measurement. This is not possible on smaller systems ( $\leq 5.0\text{m}$ ) as the measured  $P_{\text{delta}}$  cannot be accurately determined with enough precision. This leaves the moon as the only other possible celestial source of RF emissions for smaller antenna systems.

Some considerations must be taken into account when choosing an RF source for G/T calculation. Using the Sun as an RF source can yield an accurate G/T measurement for a parabolic antenna system of almost any size (1.5m and above) at any frequency (L-Band and above) commonly used in satellite communications. Conversely, the Moon is a much weaker source of RF emissions. Most of the Moon's radiation is a combination of reflected RF emissions from the Sun and other sources. As a result, a larger antenna system and higher frequencies are required for an accurate G/T measurement. The table below illustrates common parabolic antenna sizes and operating frequency bands. This table shows at what frequencies certain sized antennas should be able to accurately measure G/T using the Moon. A slightly smaller system may be able to obtain an accurate result if a Cassegrain reflector design is utilized. However, this has not been verified and is outside the scope of this paper.

**Table 1 – Moon as RF Source Antenna Sizes/Freq.**

Size	Frequency Band					
	L	S	C	X	Ku	Ka
1.5 m (5 ft.)	x	x	x	✓	✓	✓
1.8 m (6 ft.)	x	x	x	✓	✓	✓
2.4 m (8 ft.)	x	x	✓	✓	✓	✓
3.0 m (10 ft.)	x	✓	✓	✓	✓	✓
3.7 m (12 ft.)	✓	✓	✓	✓	✓	✓
5.0 m (16.4 ft.)	✓	✓	✓	✓	✓	✓
x	Unable to accurately measure at given frequency					
✓	Accurate measurement “may be” possible					
✓	Able to accurately measure at given frequency					

If the Sun is chosen as the source of RF emissions, the following empirical formula for calculating G/T is used:

$$\frac{G}{T} = 10 \log_{10} \left[ \frac{8\pi k(y-1)}{S_0 \lambda^2 C \alpha} \right] \quad (1)$$

where

- $\frac{G}{T}$  Calculated  $\frac{G}{T}$  of antenna system, dB/K
- $k$  Boltzmann Constant =  $1.38 \times 10^{-23} \text{ W/K/Hz}$
- $y$  Delta of source noise power density to cold sky power density, linear no units
- $S_0$  Solar Flux Density, expressed in Solar Flux Units (SFU)  $10^{-22} \text{ W/m}^2\text{Hz}$
- $\lambda$  Wavelength in meters
- $C$  Beam Correction Factor, linear no units
- $\alpha$  Atmospheric Attenuation at elevation angle, linear no units

If the Moon is chosen as the source of RF emissions, the same formula may be used. The value for the Solar Flux Density ( $S_0$ ) will be replaced with a calculated value for Lunar Flux Density ( $L_0$ ) using an algorithm that will be described later in this paper:

$$\frac{G}{T} = 10 \log_{10} \left[ \frac{8\pi k(y-1)}{L_0 \lambda^2 C \alpha} \right] \quad (2)$$

where

- $L_0$  Lunar Flux Density, expressed in Solar Flux Units (SFU)  $10^{-22} \text{ W/m}^2\text{Hz}$

This paper will walk the reader through making each of the calculations necessary to calculate the G/T of an antenna system using either the Sun or the Moon as an RF source. Later sections of this paper will show example G/T calculations using the Sun and the Moon as an RF source and compare the results.

## RECEIVED POWER DELTA (Y)

The value  $y$  that is required for G/T calculation represents the difference of received power from the RF source versus cold sky. This value is obtained by the same means regardless of the RF emissions source chosen. The antenna should be pointed at the RF emissions source and a received power level recorded in decibels ( $P_{source}$ ). Then the antenna should be pointed to cold sky (at the same elevation angle) and again the received power level recorded in decibels ( $P_{csky}$ ). The difference between these two values is known as  $P_{delta}$ , which is then taken out of the decibel scale to obtain a linear value:

$$P_{delta} = |P_{source} - P_{csky}| \quad (3)$$

$$y = 10^{\frac{P_{delta}}{10}} \quad (4)$$

where

- $P_{delta}$  Difference between system noise power density when the antenna is looking at the RF source and cold sky in dB
- $P_{source}$  Power in dB received by the antenna system when pointed at the RF source
- $P_{csky}$  Power in dB received by the antenna system when pointed at cold sky, at the same elevation as  $P_{source}$
- $y$  Linear representation of  $P_{delta}$

For best results, the calculated value for  $P_{delta}$  should be 1.0 dB or greater. Acceptable results can be obtained with a minimum value of 0.5 dB. Anything below this can cause the G/T calculation to return unpredictable results that can be outside the accepted margin of error. It is important to note that the accuracy of this measurement is very important. This  $y$  value has the greatest impact of any variable on the overall G/T calculation.

## WAVELENGTH ( $\lambda$ )

Obtaining the wavelength of the G/T measurement frequency is a simple conversion using the following empirical formula:

$$\lambda = c/f \quad (5)$$

where

- $\lambda$  Wavelength of the measurement frequency in meters
- $c$  Speed of light (299 792 458 m/s)
- $f$  Measurement frequency in Hertz (Hz)

## SOLAR FLUX DENSITY ( $S_0$ )

Solar Flux Density is the amount of solar energy per unit area on a sphere centered at the Sun at a distance in  $W/m^2$ . This value can be obtained by one of several different methods.

First, an antenna system that has been configured for radio telescope use can be used to obtain a Solar Flux Density measurement directly. This is the most accurate method of obtaining this value; however, it is not a practical method in most situations.

This value can be mathematically approximated as well. However, the accuracy of this approximation can vary substantially. Many factors play into the amount of solar flux that reaches the Earth's surface such as, atmospheric conditions, location on Earth's surface, receive frequency, and solar brightness temperature. The solar brightness temperature can be calculated from its radio emissions measured at the receive frequency using Rayleigh-Jeans approximation for blackbody radiation at radio frequencies [1]. Once a value for solar brightness temperature is known the following equation can be used to approximate solar flux density [1].

$$F_s = 2k_B T_d \pi \left(\frac{\alpha\pi}{180}\right)^2 \lambda^{-2} \quad (6)$$

where

- $F_s$  Solar Flux Density, Jansky  $10^{-26} W/m^2 Hz$
- $k_B$  Boltzmann Constant =  $1.38 \times 10^{-23} W/K/Hz$
- $T_d$  Solar Brightness Temperature in Kelvin
- $\alpha$  Angular Radius of the Sun in Degrees =  $0.25^\circ$
- $\lambda$  Wavelength in meters

The resulting value for  $F_s$  must then be converted to Solar Flux Units (SFU) by shifting the decimal to achieve  $10^{-22}$ . This approximation is highly dependent on the accuracy of the solar brightness temperature value which is a function of the receive frequency. This value can be measured with appropriate equipment or it can be calculated using a rather lengthy algorithm which is outside the scope of this paper [2].

The industry standard method for obtaining a value for solar flux density when a direct measurement is not possible is to interpolate a value for your desired frequency from the table of measured values published by the National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center. NOAA has stations scattered across the Earth that measure solar flux density at local noon at 9 different frequencies from 245 MHz to 15400 MHz. This table can be accessed via:

<http://services.swpc.noaa.gov/text/current-space-weather-indices.txt>

One thing to consider when using the NOAA provided Solar Flux data is the range of frequencies in which it is measured. This method will yield accurate results for measurement frequencies that are within the range of measured solar flux's (245 to 15400 MHz). More and more systems are moving to higher frequency ranges such as Ka or Ku band which is well outside this range. Using this method to interpolate a Solar Flux value at Ka frequencies has been tested and has yielded expected G/T results, but the error introduced into the resulting G/T calculation by the interpolated Solar Flux value has not been determined.

	Learmonth 0400	San Vito 1000	Sag Hill 1700
245	18	18	-1
410	44	-1	-1
610	53	-1	-1
1415	62	62	-1
2695	96	80	-1
2800	-1	-1	-1
4995	134	126	-1
8800	249	261	-1
15400	523	529	-1

**Figure 1 – Example NOAA Solar Flux Indices Data**

Above is an example of a NOAA table for values measured at 0400, 1000, and 1700 hours GMT. The column on the far left is the measurement frequency in MHz and the values are given in Solar Flux Units (SFU). A '-1' means that a value has not been measured/recorded for that time and frequency. From this table it is easy to interpolate a value for solar flux density using the values in the column representing the time closest to when the measurement was taken. Equation 7 shows the calculation for the Interpolation Exponent which will then be used in equation 8 to determine an interpolated value for Solar Flux in the units of SFU.

$$I_e = \frac{\log_{10} \frac{f_{GT}}{f_2}}{\log_{10} \frac{f_1}{f_2}} \quad (7)$$

where

- $I_e$  Interpolation Exponent
- $f_{GT}$  G/T Measurement Frequency
- $f_1$  Flux Measurement Frequency 1
- $f_2$  Flux Measurement Frequency 2

$$S_0 = f_{m2} \left( \frac{f_{m1}}{f_{m2}} \right)^{I_e} \quad (8)$$

where

$S_0$	Solar Flux Density (Interpolated) in SFU
$f_{m1}$	Flux Measurement at $f_1$
$f_{m2}$	Flux Measurement at $f_2$
$I_e$	Interpolation Exponent

This method is not the most accurate method however it is the most commonly used. The accuracy of this method greatly depends on several factors such as latitude and longitude variations of the testing system to the measurement location, weather variations between the two locations, and atmospheric differences. This method however is considered by the industry as being accurate enough as to not cause the calculated G/T to be outside the domain of acceptable error.

### LUNAR FLUX DENSITY ( $L_0$ )

If the Moon is chosen as the source for RF emissions, then the Lunar Flux Density reaching the Earth must be calculated. This is not as simple as it is when using the Sun as an RF source because there isn't a database of measured Lunar Flux values to interpolate from. This means that the value must be calculated.

Much like calculating Solar Flux Density, Lunar Flux Density is dependent on many different variables such as lunar brightness temperature, measurement frequency, and angular diameter of the Moon. The Moon however, is a much less volatile radiator than the Sun. This means that the accuracy of these calculations is much greater than what can be achieved with the Sun. The following is an expression for Lunar Flux Density which is accurate to  $\pm 4.34$  percent [3].

$$L_0 = 7.349 f^2 T d^2 \quad (9)$$

where

$L_0$	Lunar Flux Density in Jansky's ( $10^{-26}$ W/Hz/m <sup>2</sup> )
$f$	G/T Measurement Frequency in GHz
$T$	Average Lunar Brightness Temperature in Kelvin
$d$	Angular Diameter of the Moon in Degrees

This expression's error could be reduced to as little as  $\pm 1$  percent if the yearly variation in the solar insolation at the lunar surface due to the Earth's eccentric orbit were taken into account. For the purposes of calculating G/T, this error is acceptable and does not require the extra rather extensive calculations. The resulting value  $L_0$  will need to be converted to SFU by multiplying  $L_0$  by  $10^4$ .

Lunar brightness temperature is a function of measurement frequency, lunar phase angle, and lunar phase lag. The average temperature is used due to the complexity in calculating a true value. A true lunar brightness temperature would need to compensate for solar radiation changes, orbit shifts, distance variations between the Sun, Moon, Earth, and lunar surface composition. The National Bureau of Standards has published this expression for the average lunar brightness temperature with an error of  $\pm 0.18$  percent [3].

$$T = T_0 \left[ 1 - \left( \frac{T_1}{T_0} \right) \cos(\phi - \psi) \right] \quad (10)$$

where

$T$	Average Lunar Brightness Temperature, Kelvin
$T_0$	$207.7 + \frac{24.43}{f}$
$\frac{T_1}{T_0}$	$0.004212 f^{1.224}$
$f$	Measurement Frequency in GHz
$\phi$	Lunar Phase Angle in degrees (0 at New Moon)
$\psi$	Lunar Phase Lag in degrees = $\frac{43.83}{1+0.0109f}$

The algorithms required to calculate the current Lunar Phase Angle with a high degree of accuracy have been published by Jean Meeus [4]. The equation for calculating phase angle is:

$$\phi = \tan^{-1} \left( \frac{R \sin \psi}{\Delta - R \cos \psi} \right) \quad (11)$$

where

$\phi$	Lunar Phase Angle in degrees
$R$	Geocentric Distance from Earth to the Sun in km
$\Delta$	Geocentric Distance from Earth to the Moon in km
$\psi$	Geocentric Elongation of the Moon

Algorithms for calculating the geocentric distances and elongation are also given by Meeus [4]. Many of the calculations needed for determining the position of the Moon have been omitted from this paper but are available in the Meeus [4] text.

The angular diameter of the Moon is the apparent lunar diameter as seen by an observer on the Earth's surface. This value in degrees can be calculated using the following formula [3]:

$$d = \frac{0.5182}{\frac{r_0/a}{60.268} - 0.0166 \sin(elt)} \quad (12)$$

where

- $d$  Angular Lunar Diameter in degrees
- $r_0$  Geocentric Earth-Moon Separation in Kilometers
- $a$  Equatorial Radius of the Earth in Kilometers
- $el$  Elevation Angle of the Lunar Center in degrees

Values for the variables  $r_0$  and  $el$  can be obtained using the algorithms published by Meeus [4]. The generally accepted value for the Equatorial Radius of the Earth ( $a$ ) is 6378.1366 km.

The values from equations 10, 11, and 12 can now be inserted into equation 9 and a value for Lunar Flux Density calculated. The resulting value from equation 9 will be in Jansky's ( $10^{-26}$  W/Hz/m<sup>2</sup>). Multiply this value by  $10^4$  to obtain the correct units of SFU ( $10^{-22}$  W/Hz/m<sup>2</sup>).

### BEAM CORRECTION FACTOR (C)

The beam correction factor cited above has been introduced to correct the G/T measurement for errors deriving from the size of the RF source. This is especially important when using the Moon as an RF source due to its large apparent size when seen by the antenna. Since the Moon's diameter is in the neighborhood of the antenna pattern null, the antenna sees the moon not only through the pattern peak but also through points of less than maximum gain, providing receive power density less than that which would be produced if the Moon were a point source of equal brightness [5]. The expression for calculating the beam correction factor follows:

$$C = \frac{1 - \exp\left(-\left(\frac{r}{B_w}\right)^2 * \log_{10}(2)\right)}{\left(\frac{r}{B_w}\right)^2 * \log_{10}(2)} \quad (13)$$

where

- $C$  Beam Correction Factor
- $r$  Effective RF Diameter (Sun or Moon) in degrees
- $B_w$  Antenna Beamwidth in degrees

Effective RF Diameter is a function of Beamwidth ( $B_w$ ). The expression for Effective RF Diameter follows:

$$r = B_{HPBW} [1.24 - 0.162 \log_{10} f_{GT}] \quad (14)$$

where

- $r$  Effective RF Diameter in degrees
- $B_{HPBW}$  Apparent Angular Diameter in degrees
- $f_{GT}$  G/T Measurement Frequency in GHz

The apparent angular diameter used in equation 14 is the angular diameter of the object from an observer on the Earth's surface. This is also known as optical HPBW. For the Sun, this value is typically defined as 0.525 degrees. If using the Moon as the RF source, the Lunar Angular Diameter ( $d$ ) calculated in equation 12 is used here.

The value for beamwidth in degrees is calculated as follows:

$$B_w = 68 * \frac{\lambda}{D} \quad (15)$$

where

- $B_w$  Beamwidth in degrees
- $\lambda$  Wavelength in meters
- $D$  Antenna Diameter in meters

The values obtained from equation 14 and 15 can be inserted into equation 13 and that resulting value can be used in equation 2.

### ATMOSPHERIC ATTENUATION ( $\alpha$ )

Atmospheric attenuation is defined as the reduction in power density of an electromagnetic wave as it propagates through space, specifically referring to losses due to atmospheric composition, elevation angle, clouds, rain, and barometric pressure. It is a general practice to attempt measuring G/T only on days with clear skies to minimize the effects of clouds and rain. Therefore, the expressions in this paper will not compensate for clouds and rain. If implementation of a model to compensate for these factors is desired, the International Telecommunications Union (ITU) has several papers that describe the lengthy algorithms required to create these models [6] and [7].

The expressions used to calculate losses due to atmospheric attenuation are the same whether using the Sun or the Moon as an RF source and are based on Annex 2 of ITU-R P.676-11 [8]. This algorithm provides a simplified approximation method to estimate the specific attenuation due to dry air and water vapor that is applicable in the frequency range 1 – 350 GHz. The absolute difference between the results of these algorithms and the line-by-line calculation provided in Annex 1 of ITU-R P.676-11 [8] is generally less 10% for oxygen and 5% for water vapor. These expressions, while simplified are rather large and will not be covered in full detail in this paper. For an elevation angle,  $\phi$ , between 5 and 90 degrees, the path attenuation is obtained using the cosecant law, as follows:

$$A = \frac{\gamma_o h_o + \gamma_w h_w}{\sin \varphi} \quad (16)$$

where

$A$	Total Slant Path Attenuation, dB
$\gamma_o$	Specific Attenuation of Oxygen, dB/km
$h_o$	Equivalent Height of Dry Air, km
$\gamma_w$	Specific Attenuation of Water Vapor, dB/km
$h_w$	Equivalent Height of Water Vapor, km
$\varphi$	Elevation Angle, degrees

Following the algorithm presented in Annex 2 of ITU-R P.676-11 [8], the specific attenuation due to dry air and water vapor, from sea level to an altitude of 10 km, can be estimated with good agreement to the line-by-line calculation presented in Annex 1 of ITU-R P.676-11 [8]. The equations for the specific attenuations are given as:

$$\gamma_o = 0.1820f [\sum_{i(\text{Oxygen})} S_i F_i + N_D''(f)] \quad (17)$$

where

$\gamma_o$	Dry Air Specific Attenuation in dB/km
$f$	Measurement Frequency in GHz
$S_i$	Line Strength of $i^{\text{th}}$ Oxygen Line
$F_i$	Oxygen Line Shape Factor
$N_D''(f)$	Dry Air Continuum

and

$$\gamma_w = 0.1820f [\sum_{i(\text{Water Vapor})} S_i F_i] \quad (18)$$

where

$\gamma_w$	Water Vapor Specific Attenuation in dB/km
$f$	Measurement Frequency in GHz
$S_i$	Line Strength of $i^{\text{th}}$ Water Vapor Line
$F_i$	Water Vapor Line Shape Factor

The equations for calculating the line strength of oxygen and water vapor are as follows:

$$S_i = a_1 \times 10^{-7} p \theta^3 [a_2(1 - \theta)] \quad \text{for Oxygen} \quad (19)$$

$$S_i = b_1 \times 10^{-1} e \theta^{3.5} [b_2(1 - \theta)] \quad \text{for Water Vapor}$$

where

$S_i$	Line Strength
$p$	Dry Air Pressure in hPa
$e$	Water Vapor Partial Pressure in hPa
$\theta$	$300/T$
$T$	Temperature, Kelvin
$a_{\#}/b_{\#}$	Spectroscopic Data for Attenuation, Table provided in ITU-R P.676-11

If available, local altitude profiles of  $p$ ,  $e$  and  $T$  (e.g. using radiosondes) should be used. In the absence of local information, a reference standard atmosphere given in Recommendation ITU-R P.835 should be used. (Note that where total atmospheric attenuation is being calculated, the same-water vapor partial pressure is used for both dry-air and water-vapor attenuations.) The water-vapor partial pressure,  $e$ , at an altitude may be obtained from the water-vapor density,  $\rho$ , and the temperature,  $T$ , at that altitude using the expression:

$$e = \frac{\rho T}{216.7} \quad (20)$$

where

$e$	Water Vapor Partial Pressure in hPa
$\rho$	Water Vapor Density, $7.5 \text{ g/m}^3$ typical
$T$	Temperature, Kelvin

The line shape factor is given by:

$$F_i = \frac{f}{f_i} \left[ \frac{\Delta f - \delta(f_i - f)}{(f_i - f)^2 + \Delta f^2} + \frac{\Delta f - \delta(f_i + f)}{(f_i + f)^2 + \Delta f^2} \right] \quad (21)$$

where

$F_i$	Line Shape Factor
$f_i$	Spectroscopic Data for Attenuation, Table provided in ITU-R P.676-11
$f$	Measurement Frequency in GHz
$\Delta f$	Width of the Line
$\delta$	Correction Factor

Line Width

$$\Delta f = a_3 \times 10^{-4} (p \theta^{(0.8 - a_4)} + 1.1 e \theta) \quad \text{for Oxygen} \quad (22)$$

$$\Delta f = b_3 \times 10^{-4} (p \theta^{b_4} + b_5 e \theta^{b_6}) \quad \text{for Water Vapor}$$

Correction Factor

$$\delta = (a_5 + a_6 \theta) \times 10^{-4} (p + e) \theta^{0.8} \quad \text{for Oxygen} \quad (23)$$

$$\delta = 0 \quad \text{for Water Vapor}$$

As stated earlier, the values for  $f_i$ ,  $a$ , and  $b$  are all given in a summation table presented in Annex 1 of ITU-R P.676-11.

The dry air continuum arises from the non-resonant Debye spectrum of oxygen below 10 GHz and a pressure-induced nitrogen attenuation above 100 GHz and is calculated as follows:

$$N_D''(f) = fp\theta^2 \left[ \frac{6.14 \times 10^{-5}}{d \left[ 1 + \left( \frac{f}{d} \right)^2 \right]} + \frac{1.4 \times 10^{-12} p \theta^{1.5}}{1 + 1.9 \times 10^{-5} f^{1.5}} \right] \quad (24)$$

where

$N_D''(f)$  Dry Air Continuum  
 $d$  Width Parameter for the Debye spectrum

Width Parameter for the Debye spectrum:

$$d = 5.6 \times 10^{-4} (p + e) \theta^{0.8} \quad (25)$$

For dry air, the equivalent height is given by:

$$h_o = \frac{6.1}{1 + 0.17r_p^{-1.1}} (1 + t_1 + t_2 + t_3) \quad (26)$$

where

$$t_1 = \frac{4.64}{1 + 0.066r_p^{-2.3}} \exp \left[ - \left( \frac{f - 59.7}{2.87 + 12.4 \exp(-7.9r_p)} \right)^2 \right] \quad (27)$$

$$t_2 = \frac{0.14 \exp(2.12r_p)}{(f - 118.75)^2 + 0.031 \exp(2.2r_p)} \quad (28)$$

$$t_3 = \frac{0.0114}{1 + 0.14r_p^{-2.6}} f \frac{-0.0247 + 0.0001f + 1.61 \times 10^{-6} f^2}{1 - 0.0169f + 4.1 \times 10^{-5} f^2 + 3.2 \times 10^{-7} f^3} \quad (29)$$

with the constraint that:

$$h_o \leq 10.7r_p^{0.3} \quad \text{when} \quad f < 70 \text{ GHz}$$

For water vapor, the equivalent height is given by:

$$h_w = 1.66(1 + a + b + c) \quad (30)$$

where

$$a = \frac{1.39\sigma_w}{(f - 22.235)^2 + 2.56\sigma_w} \quad (31)$$

$$b = \frac{3.37\sigma_w}{(f - 183.31)^2 + 4.69\sigma_w} \quad (32)$$

$$c = \frac{1.58\sigma_w}{(f - 325.1)^2 + 2.89\sigma_w} \quad (33)$$

for  $f \leq 350 \text{ GHz}$

$$\sigma_w = \frac{1.013}{1 + \exp[-8.6(r_p - 0.57)]} \quad (34)$$

where

$$r_p = \frac{p + e}{1013.25} \quad (35)$$

Once values have been obtained for equations 17, 18, 26, and 30. These values can be entered in to equation 16 and a value for slant path atmospheric attenuation can be determined. Ensure before using this value in the G/T equation, that the value is taken out of the dB scale and converted to a linear value.

## RESULTS

In the process of developing this algorithm and writing this paper, numerous tests were run on various system sizes at various frequencies to determine the validity of the algorithm. Overall, using the Moon as an RF source, this algorithm has proven to yield a result within 5% of the result when compared to using the Sun as an RF source. This is assuming the same frequency, elevation, and antenna system. Unfortunately, it is not possible to determine exactly which RF source algorithm will yield a result closest to the “true” G/T of an antenna, unless an accurate Solar Flux measurement can be obtained at the measurement location. However, due to the nature of Lunar Flux Density and its method of calculation, it can be observed that using the Moon as an RF source will yield more stable and consistent results. With that being said, it is logical to assume that a G/T measurement using the Moon as an RF source could be considered more accurate than that of a Sun based measurement.

As stated before, many testing iterations of both these algorithms were performed using systems of varying size (1.8 – 5.0 m), frequency (L, S, X, Ka Bands), and elevations. The results of all these tests will not be presented in this paper. Instead, an example calculation will be presented using both RF sources on the same antenna system in the same location to show the validity of algorithm.

**Table 2 – Test Antenna System Parameters**

Test Antenna System	
Manufacturer	Orbital Systems, Ltd.
Model	3.0TSXS2-3.7m
Reflector Diameter	3.66 m (12 feet)
Frequency	8200.0 MHz (X-band)
Date	2018-05-22
Time	21-50 UTC
Weather	Clear Sky

**Table 3 – Spectrum Analyzer Settings**

Spectrum Analyzer Settings	
Frequency	1250 MHz IF (8200 MHz RF)
Span	2 MHz
Resolution Bandwidth	1 MHz
Video Bandwidth	100 Hz
Scale	2 dB/division
Amplitude	1 division below top of scale
Averaging	On (100 samples)

**Table 4 – Sun Based G/T Result**

<b>G/T Measurement, RF Source: Sun</b>		
<i>Measurement</i>	<i>Unit</i>	<i>Result</i>
Frequency	MHz	8200.0
Elevation Angle	degrees	41.22
Flux Measurement Freq. 1	MHz	4995.0
Flux Measurement 1	SFU	109.0
Flux Measurement Freq. 2	MHz	8800.0
Flux Measurement 2	SFU	235.0
Flux Interpolation Exponent	No Units	0.125
Solar Flux Density	SFU	213.532
Wavelength	meters	0.037
Beamwidth	degrees	0.672
Zenith Attenuation	dB	0.046
Atmospheric Attenuation	dB	0.069
Sun Optical HPBW	degrees	0.525
Sun Effective RF Diameter	degrees	0.573
Beam Correction Factor	No Units	0.786
$P_{sun}$	dBm	-51.45
$P_{csky}$	dBm	-68.12
$P_{delta}$	dBm	16.67
$y$	No Units	46.42
G/T	dB/K	<b>28.53</b>

Table 4 shows an actual G/T measurement using the Sun as an RF source performed with the antenna system shown in Table 2. All of the calculated values in table 4 were calculated using the algorithm presented in this paper.

**Table 5 – Moon Based G/T Result**

<b>G/T Measurement, RF Source: Moon</b>		
<i>Measurement</i>	<i>Unit</i>	<i>Result</i>
Frequency	MHz	8200.0
Elevation Angle	degrees	36.48
Lunar Ground Station Distance	km	371963.3
Lunar Semidiameter	degrees	0.268
Lunar Angular Diameter	degrees	0.540
Surface Radio Temperature	K	201.740
Lunar Phase Angle	degrees	80.160
Lunar Illumination	%	58.550
Wavelength	meters	0.037
Beamwidth	degrees	0.67
Lunar Flux Density	SFU	2.86
Zenith Attenuation	dB	0.047
Atmospheric Attenuation	dB	0.080
Beam Correction Factor	No Units	0.81
$P_{moon}$	dBm	-65.87
$P_{csky}$	dBm	-68.11
$P_{delta}$	dBm	2.24
$y$	No Units	1.67
G/T	dB/K	<b>28.87</b>

The above table shows the same G/T measurement using the Moon as an RF source. Notice that the same system was used on the same day at the same time in the same location and the elevation angle is roughly (< 5° difference) the same.

## DISCUSSION

As can be seen from the result tables presented above, the difference in G/T between the two RF sources is 0.34 dB/K. Repeated tests have shown that these two RF sources will yield results that vary by less than 5%. Since the algorithm presented in this paper that uses the Sun as an RF source is considered an industry standard method, it is logical to assume that the algorithm using the Moon as an RF source is also accurate.

All of the equations presented in this paper can be easily implemented in software in one's programming language of choice.

One important thing to note when implementing this algorithm is that the input values to equations 1 and 2 are linear. Therefore, atmospheric attenuation and the  $y$  value must be taken out of the logarithmic dB scale and entered into the equation as a linear value. All of the other values can be entered as is, in whatever units that they are calculated in.

The primary thing to consider when attempting to use the Moon as an RF source is the  $P_{delta}$  value. The values for  $P_{moon}$  and  $P_{csky}$  are measured values with a spectrum analyzer. With that in mind, the measured values in dBm are extremely low and close together. There needs to be a minimum  $P_{delta}$  value of 0.5 dBm for the measurement to be valid. Anything less than that value is difficult to measure with precision and will mostly consist of noise. A value entered into the equation of < 0.5 dBm will yield a result that appears correct, however, the measured difference between  $P_{moon}$  and  $P_{csky}$  cannot be determined with enough precision due to the limitations of spectrum analyzers. As a result, the final G/T value obtained cannot be trusted as being accurate.

## CONCLUSION

This paper has presented changes to the current industry standard algorithm for calculating antenna G/T to allow the use of the Moon as an RF source. These changes have been shown to result in values within 5% of the values obtained when compared to using the Sun as an RF source. It can also be concluded that with the greater stability in Lunar Flux Density calculations versus an interpolated Solar Flux Density, that using the Moon as an RF source could yield a more accurate result.

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